WEEKLY TEST TARGET JEE MATHEMATICS SOLUTION 22 SEPT 2019

61. (d) Here the centre of circle $(3, -1)$ must lie on the line $x + 2by + 7 = 0$.

Therefore, $3 - 2b + 7 = 0 \Rightarrow b = 5$.

62. (a) Radius of circle is $\left|\frac{2+3-4}{\sqrt{5}}\right| = \frac{1}{\sqrt{5}}$

Therefore, equation is $(x-1)^2 + (y+3)^2 = \frac{1}{5}$

or $x^2 + y^2 - 2x + 6y + 1 + 9 = \frac{1}{5}$

or $5x^2 + 5y^2 - 10x + 30y + 49 = 0$.

63. (b) Centre of circle = Point of intersection of diameters = (1, –1)

Now area $= 154 \Rightarrow \pi r^2 = 154 \Rightarrow r = 7$

Hence the equation of required circle is

$(x-1)^2 + (y+1)^2 = 7^2 \Rightarrow x^2 + y^2 - 2x + 2y = 47$.

64. (b) Centre (1, 2) and since circle touches x-axis, therefore, radius is equal to 2.

Hence the equation is $(x-1)^2 + (y-2)^2 = 2^2$

$\Rightarrow x^2 + y^2 - 2x - 4y + 1 = 0$.

Trick : The only circle is $x^2 + y^2 - 2x - 4y + 1 = 0$, whose centre is (1, 2).

65. (c) $2\sqrt{g^2 - c} = 2a$ ....(i)

$2\sqrt{f^2 - c} = 2b$ ....(ii)

On squaring (i) and (ii) and then subtracting (ii) from (i), we get $g^2 - f^2 = a^2 - b^2$.

Hence the locus is $x^2 - y^2 = a^2 - b^2$.

66. (b) The diameter of the circle is perpendicular distance between the parallel lines (tangents) $3x - 4y + 4 = 0$ and $3x - 4y - \frac{7}{2} = 0$ and so it is equal to $\frac{4}{\sqrt{9+16}} + \frac{7/2}{\sqrt{9+16}} = \frac{3}{2}$. Hence radius is $\frac{3}{4}$.

67. (a) Circle is $x^2 + y^2 - 2x - 2y + 1 = 0$ as centre is (1, 1) and radius = 1.

68. (c) Centre is (2, 3). One end is (3, 4).

P_2 divides the join of P_1 and O in ratio of 2 : 1.

Hence P_2 is $\left(\frac{4-3}{2-1}, \frac{6-4}{2-1}\right) \equiv (1, 2)$.

69. (c) In the equation of circle, there is no term containing xy and coefficient of x^2 and y^2 are equal. Therefore $2 - q = 0 \Rightarrow q = 2$ and $p = 3$.

70. (c) Here $2\sqrt{g^2 - c} = 2a \Rightarrow g^2 - a^2 - c = 0$ .....(i)

and it passes through (0, b), therefore

$b^2 + 2fb + c = 0$ ....(ii)

On adding (i) and (ii), we get $g^2 + 2fb = a^2 - b^2$

Hence locus is $x^2 + 2by = a^2 - b^2$.

71. (b) First find the centre. Let centre be (h, k), then

$\sqrt{(h-2)^2 + (k-3)^2} = \sqrt{(h-4)^2 + (k-5)^2}$ ....(i)

and $k - 4h + 3 = 0$ ....(ii)

From (i), we get $-4h - 6k + 8h + 10k = 16 + 25 - 4 - 9$

or $4h+4k-28=0$ or $h+k-7=0$ ....(iii)

From (iii) and (ii), we get (*h, k*) as (2, 5). Hence centre is (2, 5) and radius is 2. Now find the equation of circle.

Trick : Obviously, circle $x^2+y^2-4x-10y+25=0$ passes through (2, 3) and (4, 5).

72. (b) Centre is $(-4,-5)$ and passes through (2, 3).

73. (c) Equation of circle passing through (0, 0) is

$x^2+y^2+2gx+2fy=0$ ....(i)

Also, circle (i) is passing through (0, *b*) and (*a, b*)

$\therefore\ f=-\dfrac{b}{2}$ and $a^2+b^2+2ag+2\left(-\dfrac{b}{2}\right)b=0$

$\Rightarrow g=-\dfrac{a}{2}$

Hence the equations of circle is, $x^2+y^2-ax-by=0$.

74. (a) Equation of circle concentric to given circle is $x^2+y^2-6x+12y+k=0$ ....(i)

Radius of circle (i) $=\sqrt{2}$ (radius of given circle)

$\Rightarrow \sqrt{9+36-k}=\sqrt{2}\sqrt{9+36-15}$

$\Rightarrow 45-k=60 \Rightarrow k=-15$

Hence the required equation of circle is

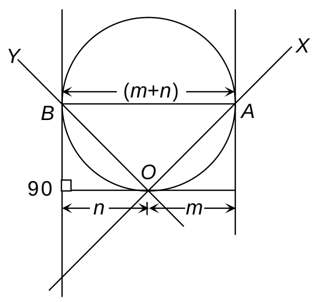
$x^2+y^2-6x+12y-15=0$.

75. (d) See condition for circle and also condition for circle to pass through origin *i.e.* origin satisfies equation of circle or $c=0$.

76. (a) Centre (3, –1). Line through it and origin is $x+3y=0$.

77. (c) As centres lie on angle bisectors of co-ordinate axes or $x=0$ and $y=0$, we get two lines which are perpendicular to each other on which the centres lie *i.e.* $x=y$ and $x=-y$ or $x^2-y^2=0$ as combined equation.

78. (b) It is clear from the figure that diameter is $m+n$.



79. (a) Let its centre be (*h, k*), then $h-k=1$ ...(i)

Also radius $a=3$

Equation is $(x-h)^2+(y-k)^2=9$

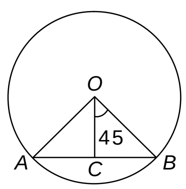
Also it passes through (7, 3)

i.e., $(7-h)^2+(3-k)^2=9$ ....(ii)

We get *h* and *k* from (i) and (ii) solving simultaneously as (4, 3). Equation is $x^2+y^2-8x-6y+16=0$.

Trick : Since the circle $x^2+y^2-8x-6y+16=0$ satisfies the given conditions.

80. (c) Let *AB* be the chord of length $\sqrt{2}$, *O* be centre of the circle and let *OC* be the perpendicular from *O* on *AB.* Then



$AB=\sqrt{2}$

$AC = BC = \frac{\sqrt{2}}{2} = \frac{1}{\sqrt{2}}$

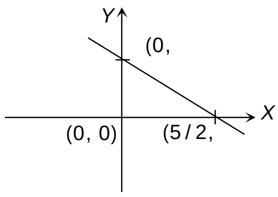
In $\Delta\, OBC, OB = BC \operatorname{cosec} 45^o = \frac{1}{\sqrt{2}}.\sqrt{2} = 1$

$\therefore$ Area of the circle $= \pi(OB)^2 = \pi$.

81. (b) Given, triangle formed by the lines $x = 0$, $y = 0$, $2x + 3y = 5$, so vertices of the triangle are (0, 0), (5/2, 0) and (0, 5/3).

Since circle is passing through (0, 0).

$\therefore$ Equation of circle will be $x^2 + y^2 + 2gx + 2fy = 0$ ..(i)



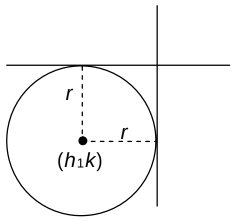
Also, circle is passing through (5/2, 0) and (0, 5/3)

So, $g = -5/4$, $f = -5/6$.

Put the values of *g* and *f* in equation (i).

After solving, we get $6(x^2 + y^2) - 5(3x + 2y) = 0$, which is the required equation of the circle.

82. (d) Since circle touches the co-ordinate axes in III quadrant.



$\therefore$ Radius $= -h = -k$. Hence $h = k = -5$

$\therefore$ Equation of circle is $(x+5)^2 + (y+5)^2 = 25.$

83. (b) $x = 2 + 3\cos\theta, y = 3\sin\theta - 1$

$x^2 + y^2 = 4 + 9\cos^2\theta + 12\cos\theta + 9\sin^2\theta + 1 - 6\sin\theta$

$= 14 + 12\cos\theta - 6\sin\theta$

$= 4(2 + 3\cos\theta) - 2(3\sin\theta - 1) + 4$

$\Rightarrow x^2 + y^2 = 4x - 2y + 4$

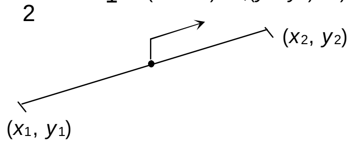
$\Rightarrow (x^2 - 4x + 4) + (y^2 + 2y + 1) = 9$

$\Rightarrow (x-2)^2 + (y+1)^2 = 9$, $\therefore$ centre is $(2, -1)$.

84. (a) x_1, x_2 are roots of $x^2 + 2x + 3 = 0$

$\Rightarrow x_1 + x_2 = -2$

$\therefore \frac{x_1 + x_2}{2} = -1$



y_1, y_2 are roots of $y^2 + 4y - 12 = 0$

$\Rightarrow y_1 + y_2 = -4 \Rightarrow \frac{y_1 + y_2}{2} = -2$

Centre of circle $\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right) = (-1,-2)$.

85. (c) The other end is $(t, 3-t)$

So the equation of the variable circle is

$(x-1)(x-t)+(y-1)(y-3+t)=0$

or $x^2+y^2-(1+t)x-(4-t)y+3=0$

$\therefore$ The centre (α, β) is given by

$\alpha = \frac{1+t}{2}, \beta = \frac{4-t}{2}$

$\Rightarrow \quad 2\alpha + 2\beta = 5$

Hence, the locus is $2x+2y=5$.

86. (b) According to the condition,

$\sqrt{(5)^2+(3)^2+2(5)+k(3)+17} = 7$

$\Rightarrow 61+3k=49 \Rightarrow k=-4$.

87. (b) Line $y=mx+c$ is tangent, if $c = \pm a\sqrt{1+m^2}$.

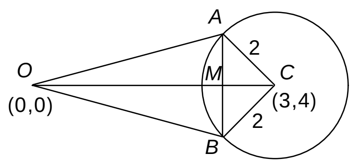
Now $lx+my+n=0$ or $y = -\frac{l}{m}x - \frac{n}{m}$ is tangent, if

$-\frac{n}{m} = \pm a\sqrt{1+\left(\frac{l}{m}\right)^2}$ or $n^2 = a^2(m^2+l^2)$.

88. (b) Here the equation of *AB* (chord of contact) is

$0+0-3(x+0)-4(y+0)+21=0$

$\Rightarrow 3x+4y-21=0$ ....(i)



CM = perpendicular distance from (3, 4) to line (i) is

$\frac{3\times3+4\times4-21}{\sqrt{9+16}} = \frac{4}{5}$

$AM = \sqrt{AC^2-CM^2} = \sqrt{4-\frac{16}{25}} = \frac{2}{5}\sqrt{21}$

$\therefore \quad AB = 2AM = \frac{4}{5}\sqrt{21}$.

89. (c) Find points of intersection by simultaneously solving for *x* and *y* from $y=mx+c$ and $x^2+y^2=a^2$ which comes out as $\left(-\frac{a^2m}{c}, \frac{a^2}{c}\right)$.

90. (b) Point is inside, outside or on the circle as S_1 is <, >, = 0. For point (– 2, 1), $S_1 < 0$.